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MODELLING OF THE MOTION OF PARTICLES IN NON-UNIFORM TURBULENT FLOW USING THE EQUATION FOR THE PROBABILITY DENSITY FUNCTION†

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A closed equation for the probability density function of the velocity of particles is obtained in explicit form using the functional-differentiation method, taking into account the non-uniformity of the velocity field of the carrying turbulent flow. A system of continual equations of the balance of mass, momentum and second moments of the velocity pulsations of the dispersed phase is constructed. The result is compared with published solutions for a uniform layer. © 1997 Elsevier Science Ltd. All rights reserved.

The interaction between particles and turbulent vortices of the carrying phase has previously [1–3] been described by the same diffusion operator in velocity space as for Brownian diffusion, while the kinetic equation for the probability density function is essentially identical with the usual Fokker–Planck equation. However, the Fokker–Planck equation only holds for modelling random processes that are δ-correlated in time, and hence in practice for describing inertial particles, the dynamic-relaxation time of which considerably exceeds the integral turbulence timescale. An equation for the probability density function in a more general form than the Fokker–Planck equation was obtained in [4–6] for modelling turbulent gas-phase fields by Gaussian random processes with known correlation functions; however these equations ignore the spatial non-uniformity of the carrying flow. The kinetic equation was constructed in [7] in implicit form, taking into account the non-uniformity of the flow, by summing the direct interactions using the method of renormalization perturbation theory.

1. THE EQUATION FOR THE PROBABILITY DENSITY FUNCTION OF THE PARTICLE VELOCITY

The motion of a single solid particle in gaseous flow is described by the following equations

$$\frac{d\mathbf{R}_p}{d\tau}=\mathbf{v}_p,\quad \frac{d\mathbf{v}_p}{d\tau}=\frac{\mathbf{u}-\mathbf{v}_p}{\tau_u}+\mathbf{F} \tag{1.1}$$

where τ is the time, $\mathbf{R}_p$ and $\mathbf{v}_p$ are the coordinate and velocity of the particle, $\mathbf{u}$ is the velocity of the carrying flow, τ_u is the particle dynamic relaxation time, and $\mathbf{F}$ is the acceleration of the external force.

Expressions (1.1) represent Langevin-type equations in which the velocity of the gas u is regarded as a random process. In order to change from a dynamic stochastic description of the individual particles to modelling of the statistical behaviour of the dispersed phase we will introduce the probability density function of the distribution over the particle coordinates and velocities

$$P=\langle p\rangle=\frac{\omega}{\Omega}\sum_p\langle\delta(\mathbf{x}-\mathbf{R}_p(\tau))\delta(\mathbf{v}-\mathbf{v}_p(\tau))\rangle \tag{1.2}$$

where ω is the volume of a particle and Ω is the spatial volume considered.

Averaging in (1.2) is carried out over the ensemble of samples of the random turbulent velocity fields of the carrying flow $\mathbf{u}(\mathbf{x},\tau)$.

Differentiating (1.2) with respect to time, taking (1.1) into account, and representing the velocity of the gas in the form of the averaged and pulsation components $\mathbf{u}=\mathbf{U}+\mathbf{u}'$, we obtain the following

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equation for the probability density function

$$\frac{\partial P}{\partial \tau}+v_k\frac{\partial P}{\partial x_k}+\frac{\partial}{\partial v_k}\left[\left(\frac{U_k-v_k}{\tau_u}+F_k\right)P\right]=-\frac{1}{\tau_u}\frac{\partial\langle u_k'p\rangle}{\partial v_k} \tag{1.3}$$

To determine the correlator $\langle u_i'p\rangle$ in (1.3) the velocity field of the carrying flow, as in [4, 5], is modelled by a Gaussian process with known autocorrelation function. Using the Furutz–Novikov formula for Gaussian random functions [8] we obtain

$$\langle u_i'p\rangle=\int\int\langle u_i'(\mathbf{x},\tau)u_k'(\mathbf{x}_1,\tau_1)\rangle\left\langle\frac{\delta p(\mathbf{x},\tau)}{\delta u_k(\mathbf{x}_1,\tau_1)}\right\rangle d\mathbf{x}_1\,d\tau_1$$
$$\left\langle\frac{\delta p(\mathbf{x},\tau)}{\delta u_k(\mathbf{x}_1,\tau_1)}\right\rangle=-\frac{\partial}{\partial x_j}\left\langle p(\mathbf{x},\tau)\frac{\delta R_{pj}(\tau)}{\delta u_k(\mathbf{x}_1,\tau_1)}\right\rangle-\frac{\partial}{\partial v_j}\left\langle p(\mathbf{x},\tau)\frac{\delta v_{pj}(\tau)}{\delta u_k(\mathbf{x}_1,\tau_1)}\right\rangle \tag{1.4}$$

To find the functional derivatives in (1.4) we use the solutions of the equations of motion of a single particle

$$R_{pi}(\tau)=\int_0^\tau v_{pi}(\tau_1)d\tau_1$$
$$v_{pi}(\tau)=\int_0^\tau\left[\frac{u_i(\mathbf{R}_p(\tau_1),\tau_1)}{\tau_u}+F_i(\mathbf{R}_p(\tau_1),\tau_1)\right]\exp\left(-\frac{\tau-\tau_1}{\tau_u}\right)d\tau_1 \tag{1.5}$$

Applying the functional-differentiation operator to (1.5) we obtain a system of integral equations for determining the functional derivatives

$$\frac{\delta R_{pi}(\tau)}{\delta u_j(\mathbf{x}_1,\tau_1)}=\delta_{ij}\left[1-\exp\left(-\frac{\tau-\tau_1}{\tau_u}\right)\right]\delta(\mathbf{x}_1-\mathbf{R}_p(\tau_1))H(\tau-\tau_1)+\int_{\tau_1}^{\tau}\left[1-\exp\left(-\frac{\tau-\tau_2}{\tau_u}\right)\right]\times$$
$$\times\frac{\partial}{\partial x_n}[u_i(\mathbf{R}_p(\tau_2),\tau_2)+\tau_uF_i(\mathbf{R}_p(\tau_2),\tau_2)]\frac{\delta R_{pn}(\tau_2)}{\delta u_j(\mathbf{x}_1,\tau_1)}d\tau_2 \tag{1.6}$$

$$\frac{\delta V_{pi}(\tau)}{\delta u_j(\mathbf{x}_1,\tau_1)}=\frac{\delta_{ij}}{\tau_u}\exp\left(-\frac{\tau-\tau_1}{\tau_u}\right)\delta(\mathbf{x}_1-\mathbf{R}_p(\tau_1))H(\tau-\tau_1)+\frac{1}{\tau_u}\int_{\tau_1}^{\tau}\exp\left(-\frac{\tau-\tau_2}{\tau_u}\right)\times$$
$$\times\frac{\partial}{\partial x_n}[u_i(\mathbf{R}_p(\tau_2),\tau_2)+\tau_uF_i(\mathbf{R}_p(\tau_2),\tau_2)]\frac{\delta R_{pn}(\tau_2)}{\delta u_j(\mathbf{x}_1,\tau_1)}d\tau_2 \tag{1.7}$$

where $H(x)$ is the Heaviside function: $H(x<0)=0, H(x>0)=1$.

To obtain explicit expressions for the functional derivatives and, correspondingly, for the correlator $\langle u_i'p\rangle$, the integral terms in (1.6) and (1.7) were eliminated [4, 5] from consideration; however, the effect of these terms may be considerable in non-uniform flows. In [9] only the integral term in (1.6) was neglected, and to determine the integral term in (1.7) an approximation was proposed which effectively takes into account the non-uniformity of the carrying flow in terms of the non-uniformity of the field of the averaged velocity of the dispersed phase.

In this paper, to solve integral equations (1.6) and (1.7) we will use an iterative method, taking the quantity $T_p\Delta U/\Delta x$ as the small parameter, where T_p is the interaction time of the particles with the energy-containing vortices of the carrying flow, ΔU is the scale of variation of the flow velocity and Δx is a characteristic spatial scale. We take the first term on the right-hand side of (1.6) as the first approximation of the solution of Eq. (1.6), which is accurate for uniform flow. The second term of the iterative expansion, which takes the non-uniformity of the flow into account up to first-order spatial derivatives, has the form

$$\frac{\delta R_{pi}(\tau)}{\delta u_j(\mathbf{x}_1,\tau_1)}=\delta_{ij}\left[1-\exp\left(-\frac{\tau-\tau_1}{\tau_u}\right)\right]\delta(\mathbf{x}_1-\mathbf{R}_p(\tau_1))H(\tau-\tau_1)+\delta(\mathbf{x}_1-\mathbf{R}_p(\tau_1))\int_{\tau_1}^{\tau}\times$$

$$\times\left[1-\exp\left(-\frac{\tau-\tau_2}{\tau_u}\right)\right]\left[1-\exp\left(-\frac{\tau_2-\tau_1}{\tau_u}\right)\right]\frac{\partial}{\partial x_j}[u_i(\mathbf{R}_p(\tau_2),\tau_2)+\tau_u F_i(\mathbf{R}_p(\tau_2),\tau_2)]d\tau_2 \tag{1.8}$$

Using (1.8) we can write expression (1.7) in the form

$$\frac{\delta v_{pi}(\tau)}{\delta u_j(\mathbf{x}_1,\tau_1)}=\frac{\delta_{ij}}{\tau_u}\exp\left(-\frac{\tau-\tau_1}{\tau_u}\right)\delta(\mathbf{x}_1-\mathbf{R}_p(\tau_1))H(\tau-\tau_1)+$$
$$+\frac{1}{\tau_u}\int_{\tau_1}^{\tau}\exp\left(-\frac{\tau-\tau_1}{\tau_u}\right)\frac{\partial}{\partial x_n}[u_i(\mathbf{R}_p(\tau_2),\tau_2)+\tau_u F_i(\mathbf{R}_p(\tau_2),\tau_2)]\times$$
$$\times\left\{\delta_{jn}\left[1-\exp\left(-\frac{\tau_2-\tau_1}{\tau_u}\right)\right]\delta(\mathbf{x}_1-\mathbf{R}_p(\tau_1))H(\tau_2-\tau_1)+\right.$$
$$+\delta(\mathbf{x}_1-\mathbf{R}_p(\tau_1))\int_{\tau_1}^{\tau_2}\left[1-\exp\left(-\frac{\tau_2-\tau_3}{\tau_u}\right)\right]\left[1-\exp\left(-\frac{\tau_3-\tau_1}{\tau_u}\right)\right]\times$$
$$\left.\times\frac{\partial}{\partial x_j}[u_i(\mathbf{R}_p(\tau_3),\tau_3)+\tau_u F_i(\mathbf{R}_p(\tau_3),\tau_3)]d\tau_3\right\}d\tau_2 \tag{1.9}$$

We will introduce a two-time correlation function of the velocity pulsations of the gas along the trajectories of the particles

$$\Psi(\tau-\tau_1)=\frac{\langle u_i'(\mathbf{x},\tau)u_j'(\mathbf{R}_p(\tau_1),\tau_1)\rangle}{\langle u_i'(\mathbf{x},\tau)u_j'(\mathbf{x},\tau)\rangle} \tag{1.10}$$

Then, assuming that, in view of the rapid decrease in the function $\Psi(\xi)$ as ξ increases, the main contribution to the integrals is governed by the region with $\xi\approx 0$, and assuming the external force to be homogeneous, we can obtain from (1.4) and (1.8)–(1.10)

$$\langle u_i'p\rangle=-\langle u_i'u_k'\rangle\left(f_u\frac{\partial P}{\partial v_k}+\tau_u g_u\frac{\partial P}{\partial x_k}+\tau_u l_u\frac{\partial U_n}{\partial x_k}\frac{\partial P}{\partial v_n}+\tau_u^2 h_u\frac{\partial U_n}{\partial x_k}\frac{\partial P}{\partial x_n}+\right.$$
$$\left.+\tau_u^2 m_u\frac{\partial U_j}{\partial x_n}\frac{\partial U_n}{\partial x_k}\frac{\partial P}{\partial v_j}\right) \tag{1.11}$$

Here

$$f_u=I_0,\quad g_u=J_0-I_0,\quad l_u=g_u-I_1$$
$$h_u=J_1+I_1-2g_u,\quad m_u=J_1+2I_1+I_2-3g_u$$
$$I_n=\frac{1}{n!\tau_u^{n+1}}\int_0^\infty\Psi(\xi)\xi^n\exp\left(-\frac{\xi}{\tau_u}\right)d\xi$$
$$J_n=\frac{1}{n!\tau_u^{n+1}}\int_0^\infty\Psi(\xi)\xi^n d\xi$$

Expression (1.11) holds for a value of the time that is long compared with the Lagrange integral timescale of the turbulence T_L. The coefficients f_u, g_u, l_u, h_u, m_u in (1.11) define the degree of involvement of the particles in the macropulsation motion of the carrying flow. To calculate these coefficients we need to know the correlation function $\Psi(\xi)$, characterized by the time during which the particles interact with the energy-containing vortices of the gas $T_p = \tau_u J_0$. For non-inertial particles the interaction time T_p with the velocity fluctuations is identical with the integral Lagrange turbulence scale T_1, characterizing the decay of the energy-containing pulsations of the carrying flow with time. For inertial particles, when there is considerable averaged slipping of the particles with respect to the gaseous medium, $T_p < T_L$.

These coefficients satisfy the asymptotic relations

$$\frac{\tau_u}{T_p} \to 0: \quad f_u = 1, \quad g_u = l_u = J_0, \quad h_u = m_u = J_1 \tag{1.12}$$

$$\frac{\tau_u}{T_p} \to \infty: \quad f_u = J_0, \quad g_u = J_1, \quad l_u = J_2, \quad h_u = J_3, \quad m_u = J_4 \tag{1.13}$$

Substituting (1.11) into (1.3) we obtain a closed kinetic equation for the probability density function of the particle velocity in non-uniform turbulent flow

$$\frac{\partial P}{\partial \tau} + v_k \frac{\partial P}{\partial x_k} + \frac{\partial}{\partial v_k}\left[\left(\frac{U_k - v_k}{\tau_u} + F_k\right)P\right] = \langle u_i' u_k' \rangle \left(\frac{f_u}{\tau_u}\frac{\partial^2 P}{\partial v_i \partial v_k} + g_u \frac{\partial^2 P}{\partial x_i \partial v_k} + \right.$$
$$\left. + l_u \frac{\partial U_n}{\partial x_k}\frac{\partial^2 P}{\partial v_i \partial v_n} + \tau_u h_u \frac{\partial U_n}{\partial x_k}\frac{\partial^2 P}{\partial x_n \partial v_i} + \tau_u m_u \frac{\partial U_j}{\partial x_n}\frac{\partial U_n}{\partial x_k}\frac{\partial^2 P}{\partial v_i \partial v_j}\right) \tag{1.14}$$

The terms on the left-hand side of Eq. (1.14) describe the change with time and the convection of the probability density function in phase space x, v, while the right-hand side describes the diffusion in phase space due to interaction between the particles and turbulent vortices of continuous phase. The last three terms on the right-hand side of (1.14) are directly related to the non-uniformity of the velocity field of the carrying flow. When these terms are not present Eq. (1.14) becomes the kinetic equation for the probability density function of the particle velocity [4–6]. The effect of flow non-uniformity, by (1.12), is particularly significant for small particles ($\tau_u \ll T_p$), and by (1.13), the part played by the terms responsible for the non-uniformity for large particles ($\tau_u/T_p \gg 1$) is small.

2. THE EQUATIONS OF THE BALANCE OF MASS, MOMENTUM AND SECOND MOMENTS OF THE PULSATIONS OF THE VELOCITY OF THE DISPERSED PHASE

By integrating Eq. (1.14) over velocity space we can obtain a system of continual equations for the average characteristics (moments) of the dispersed phase. The equation of conservation of mass has the form

$$\frac{\partial \Phi}{\partial \tau} + (\Phi V_k)_{,k} = 0 \quad \left(\Phi = \int P d\mathbf{v}, \ V_i = \frac{1}{\Phi}\int v_i P d\mathbf{v}\right) \tag{2.1}$$

where Φ and V_i are the average volume concentration and average components of the velocity of the dispersed phase respectively, and the comma in front of the subscript k denotes differentiation with respect to the coordinate x_k.

The momentum balance equation can be written in the form

$$\frac{\partial V_i}{\partial \tau} + V_k V_{i,k} = -q_{ik,k} + \frac{U_i - V_i}{\tau_u} + F_i - \frac{D_{pik}}{\tau_u}(\ln \Phi)_{,k} \tag{2.2}$$

Here

$$q_{ij} = \langle v_i' v_j' \rangle = \frac{1}{\Phi}\int (v_i - V_i)(v_j - V_j) P dv$$

are the turbulent stresses in the dispersed phase due to the involvement of particles in the pulsation motion of the continuous medium. The last term in (2.2) describes the turbulent diffusion of the particles. The particle turbulent diffusion tensor is defined by the expression

$$D_{pij} = \tau_u(q_{ij} + g_u p_{ij} + \tau_u h_u p_{ik} U_{j,k}), \quad p_{ij} = \langle u_i' u_j' \rangle \tag{2.3}$$

The equation for the second moments of the velocity pulsations has the form

$$\begin{aligned}
&\frac{\partial q_{ij}}{\partial \tau} + V_k q_{ik,k} + \frac{1}{\Phi}(\Phi \langle v_i' v_j' v_k' \rangle)_{,k} = -q_{ik} V_{j,k} - q_{jk} V_{i,k} - g_u (p_{ik} V_{j,k} + p_{jk} V_{i,k}) + \\
&+ l_u (p_{ik} U_{j,k} + p_{jk} U_{i,k}) - \tau_u h_u U_{n,k} (p_{ik} V_{j,n} + p_{jk} V_{i,n}) + \\
&+ \tau_u m_u U_{n,k} (p_{ik} U_{j,n} + p_{jk} U_{i,n}) + \frac{2}{\tau_u}(f_u p_{ij} - q_{ij})
\end{aligned} \tag{2.4}$$

Equation (2.4) describes the convective and diffusion transfer, the generation of pulsations from the averaged non-uniform motion, and the generation of fluctuations resulting from the involvement of particles in the pulsation motion of the carrying flow and dissipation of turbulent energy of the dispersed phase due to the work done by the interphase interaction force. The equations of conservation of mass and momentum of the dispersed phase are identical with the corresponding equations obtained previously [4, 5], while the equation for the second moments of the velocity pulsations, like the expression for the particle diffusion tensor, contains additional terms due to the non-uniformity of the carrying flow.

In the limit of very fine particles, we obtain the following expression for the velocity of a non-inertial impurity

$$V_i = U_i - D_{ik} (\ln \Phi)_{,k} \tag{2.5}$$

By (2.5) the velocity of non-inertial particles is made up of convective and diffusion components. The turbulent diffusion tensor of a non-inertial impurity (a passive scalar), by (2.3), is equal to

$$D_{ij} = \lim_{\tau_u \to 0} D_{pij} = T_L p_{ij} + \chi p_{ij} U_{j,k}, \quad \chi = \int_0^\infty \Psi(\xi) \xi d\xi \tag{2.6}$$

From (2.1) and (2.5) we obtain the diffusion equation for a non-inertial impurity

$$\partial \Phi / \partial \tau + (\Phi U_k)_{,k} = (D_{ik} \Phi_{,k})_{,i} \tag{2.7}$$

Equation (2.7) is the usual equation of the turbulent diffusion of a passive scalar, in which the effect of the non-uniformity of the flow on the turbulent transfer mechanism is taken into account by the presence of the second term in (2.6).

3. A UNIFORM SHEAR LAYER

We will consider the motion of particles in a uniform shear layer. In this case, because of the simplicity of the flow in question, exact expressions can be obtained for the diffusion tensor and for the turbulent stresses of the dispersed phase. The flow is assumed to take place in the x direction and has a constant shear velocity in the y direction, i.e. $\gamma = dU_x/dy = \text{const}$.

The components of the turbulent diffusion tensor of a non-inertial impurity, by (2.6), in this case take the form

$$D_{xx} = T_L p_{xx} + \chi p_{xy} \gamma, \quad D_{xy} = T_L p_{xy} + \chi p_{yy} \gamma, D_{yx} = T_L p_{xy}, \quad D_{yy} = T_L p_{yy}, \quad D_{zz} = T_L p_{zz} \tag{3.1}$$

Expressions (3.1) are identical with the relations obtained previously in [10], which extend the expressions for the turbulent diffusion coefficients of a passive scalar in terms of the Lagrangian correlation moments of the velocity pulsations for uniform steady motion without shear of the mean velocity [11] to the case of the simplest shear flow with a constant shear of the mean velocity.

The components of the turbulent diffusion tensor of inertial particles (2.3) in a uniform shear layer are represented in the form

$$D_{pxx} = \tau_u(q_{xx} + g_u p_{xx}) + \tau_u^2 h_u p_{xy}\gamma, D_{pxy} = \tau_u(q_{xy} + g_u p_{xy}) + \tau_u^2 h_u p_{yy}\gamma$$
$$D_{pyx} = \tau_u(q_{xy} + g_u p_{xy}), \quad D_{pyy} = \tau_u(q_{yy} + g_u p_{yy}), D_{pzz} = \tau_u(q_{zz} + g_u p_{zz}) \tag{3.2}$$

It should be noted that the turbulent diffusion tensors of both inertial and non-inertial particles (2.3) and (2.6) are asymmetric; this can also be clearly seen from relations (3.2).

The asymmetry of the turbulent diffusion tensor of a passive scalar in a uniform shear layer has been noted previously in [10]; a similar result was obtained in [12] when analysing the dispersion of inertial particles in a random field with a constant shear of the mean velocity by using the eigenvalue method.

For flow with a uniform shear the system of differential equations for the second moments of the velocity pulsations of the dispersed phase (2.4) reduces to a system of algebraic equations, from which we obtain

$$q_{xx} = f_u p_{xx} - T_L p_{xy}\gamma_p + \tau_u l_u p_{xy}\gamma + \frac{\tau_u}{2} p_{yy}(T_L\gamma_p - \tau_u l_u\gamma)\gamma_p$$
$$q_{xy} = f_u p_{xy} - \frac{T_L}{2} p_{yy}\gamma_p + \frac{\tau_u}{2} l_u p_{yy}\gamma, \quad \gamma_p = \frac{dV_x}{dy} \tag{3.3}$$

$$q_{yy} = f_u p_{yy}, \quad q_{zz} = f_u p_{zz} \tag{3.4}$$

Expressions (3.3) take into account, in explicit form, the effect of shears of the mean velocities of the continuous and dispersed phases on the intensity of longitudinal velocity pulsations in the shear stress of the particles. As can be seen from (3.4), shear has no effect on the intensity of the velocity pulsations of the particles in the transverse and transversal directions. Expressions (3.3) and (3.4) are identical with the corresponding relations for turbulent stresses of particles in a layer with a constant shear, obtained in [13] from the continual equations of the balance of the second moments of the velocity pulsations of the dispersed phase [6].

Assuming that $\gamma_p = \gamma$, Eqs (3.3) can be represented in the form

$$q_{xx} = f_u p_{xx} - \tau_u(f_u + l_1)p_{xy}\gamma + \frac{\tau_u^2}{2}(f_u + l_1)p_{yy}\gamma^2, q_{xy} = f_u p_{xy} - \frac{\tau_u}{2}(f_u + l_1)p_{yy}\gamma \tag{3.5}$$

It can be seen from (3.3)–(3.5) that the anisotropy of the turbulent pulsations of the dispersed phase increases both as the shear of the mean velocity of the flow increases and as the particle inertia increases. It is interesting to note that in shear flow the intensity of the longitudinal velocity pulsations of fairly inertial particles may exceed the corresponding intensity of the turbulence of the carrying medium. This effect is due to the generation of particle velocity fluctuations in the longitudinal direction from the averaged motion due to velocity shear.

Using relations (3.4) and (3.5) the components of the turbulent diffusion tensor of the particles (3.2) can be written in the form

$$D_{pxx} = T_L p_{xx} + \tau_u^2(f_u + J_1 - 2J_0)p_{xy}\gamma + \frac{\tau_u^3}{2}(f_u + l_1)p_{yy}\gamma^2$$
$$D_{pxy} = T_L p_{xy} + \tau_u^2\left(\frac{3}{2}f_u + J_1 + \frac{l_1}{2} - 2J_0\right)p_{yy}\gamma \tag{3.6}$$
$$D_{pyx} = T_L p_{xy} - \frac{\tau_u^2}{2}(f_u + l_1)p_{yy}\gamma, D_{pyy} = T_L p_{yy}, \quad D_{pzz} = T_L p_{zz}$$

Formulae (3.6) agree with the relations obtained in [12, 13] for the components of the diffusion tensor of particles in a uniform shear layer.

Hence, our analysis shows that for turbulent flow with constant shear one can obtain from the kinetic equation for the probability density function (1.14) expressions for the diffusion tensor and second moments of the velocity pulsations of particles, which agree with published solutions.

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